

Status of $|V_{cb}|$ and $|V_{ub}|$ CKM matrix elements

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Abstract. We summarize the status of $|V_{cb}|$ and $|V_{ub}|$ determinations, including the long standing tension among exclusive and inclusive determinations. We also discuss B meson semi-leptonic decays to excited states of the charm meson spectrum and leptonic and semileptonic B decays into final states which include τ leptons.

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INTRODUCTION

The increasing precision in the measurements and theoretical calculations of physical observables requires an accurate knowledge of the CKM parameters. At present, V_{ud} is the best known parameter, with a relative uncertainty of the order 10^{-4} ; precise determinations of V_{cs} , of the order of 10^{-2} and less, are also available, although a slight tension arises when combining results from leptonic K decays, semileptonic K decays, and τ decays. The uncertainties on all other $|V_{ij}|$ CKM parameters range from about 2 to 7 10^{-2} ; $|V_{ub}|$ stands as having the last precise estimate, with an uncertainty reaching 10%. $|V_{cb}|$ and $|V_{ub}|$ are two fundamental parameters of the unitarity triangle analysis, which are also crucial for the identification of new physics [1]. At present, the most precise values of $|V_{cb}|$ and $|V_{ub}|$ come from inclusive and exclusive semileptonic decays. The inclusive and exclusive determinations rely on different theoretical calculations and on different experimental techniques which have, to a large extent, uncorrelated statistical and systematic uncertainties. This independence makes the comparison of $|V_{cb}|$ and $|V_{ub}|$ values from inclusive and exclusive decays an interesting test of our physical understanding. Another determination of $|V_{ub}|$ is given by the measurement of the rate of the leptonic decays $B^+ \rightarrow l^+ \nu$, provided that the B -decay constant is known from theory. This determination is disadvantaged by the helicity suppression and by the possibility of a more relevant role of new physics.

Here, we summarize significant and recent results on heavy-to-heavy and heavy-to-light semi-leptonic decays, and the status of $|V_{cb}|$ and $|V_{ub}|$ extraction¹. We also discuss B meson semileptonic decays to excited states of the charm meson spectrum and outline the status of leptonic and semileptonic B decays into τ leptons.

HEAVY-TO-HEAVY DECAYS

Exclusive decays

For negligible lepton masses ($l = e, \mu$), the differential ratios for the semi-leptonic decays $B \rightarrow D^{(*)} l \nu$ are proportional to $|V_{cb}|^2$, and can be written as

$$\frac{d\Gamma}{d\omega}(B \rightarrow D l \nu) = \frac{G_F^2}{48\pi^3} (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{\frac{3}{2}} |V_{cb}|^2 |\eta_{EW}|^2 |\mathcal{G}(\omega)|^2$$

¹ For recent reviews see e.g. Refs. [2, 3, 4, 5, 6, 7, 8], and references therein.

$$\frac{d\Gamma}{d\omega}(B \rightarrow D^* l \nu) = \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 m_{D^*}^3 \chi(\omega) (\omega^2 - 1)^{\frac{1}{2}} |V_{cb}|^2 |\eta_{EW}|^2 |\mathcal{F}(\omega)|^2 \quad (1)$$

in terms of a single form factor $\mathcal{G}(\omega)$ and $\mathcal{F}(\omega)$, for $B \rightarrow D l \nu$ and $B \rightarrow D^* l \nu$, respectively. In Eq. (1), η_{EW} is an EW enhancement factor and $\chi(\omega)$ is a phase space factor which reads

$$\chi(\omega) = (\omega + 1)^2 \left(1 + \frac{4\omega}{\omega + 1} \frac{m_B^2 - 2\omega m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2} \right) \quad (2)$$

The parameter $\omega = p_B \cdot p_{D^*} / m_B m_{D^*}$ corresponds to the energy transferred to the leptonic pair. In the heavy quark limit both form factors are related to a single Isgur-Wise function, $\mathcal{F}(\omega) = \mathcal{G}(\omega) = \xi(\omega)$, which is normalized at zero recoil, that is $\xi(\omega = 1) = 1$. Beyond heavy mass limit, non-perturbative contributions add to the unit limit terms depending on $m = m_c$ and m_b

$$\mathcal{F}(\omega = 1) = 1 + O\left(\frac{1}{m^2}\right) \quad \mathcal{G}(\omega = 1) = 1 + O\left(\frac{1}{m}\right) \quad (3)$$

The FNAL/MILC collaboration has performed the non perturbative determination of the form factor $\mathcal{F}(1)$ in the lattice unquenched $N_f = 2 + 1$ approximation [9, 10]. The FNAL/MILC collaboration uses FNAL b -quark and asqtad u, d, s valence quarks. The most recent update exploits the full suite of MILC (2+1)-flavor asqtad ensembles for sea quarks, lattice spacings as small as 0.045 fm and light-to-strange-quark mass ratios as low as 1/20 [11]. The form factor estimate is

$$\mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012 \quad (4)$$

The first error is statistical and the second one systematic. Using the previous form factor and the latest HFAG average, the following estimate for $|V_{cb}|$ can be given [11]

$$|V_{cb}| = (39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{latt}} \pm 0.19_{\text{QED}}) \times 10^{-3} \quad (5)$$

which it reported in Table 1. The central value is not very different from the central value of the 2009 determination from the same Collaboration [9], but errors are considerably reduced. The lattice QCD theoretical error is now commensurate with the experimental error, they contribute respectively for about 1.4% and 1.3%, while the QED error contributes for about 0.5%. Largest QCD errors come from discretization and are estimated taking the difference between HQET description of lattice gauge theory and QCD. Other, preliminary, values for the $B \rightarrow D^*$ form factor at zero recoil, in agreement with the value reported in (4), have also been obtained at $N_f = 2$ by using two ensembles of gauge configurations produced by the European Twisted Mass Collaboration (ETMC) [12]. At a variance with the approach used by the FNAL/MILC collaboration, in Ref. [12] form factors and then the branching ratios are determined using charmed quarks having a realistic finite mass, without recourse to the infinite mass limit.

At the current level of precision, it would be important to extend form factor calculations for $B \rightarrow D^*$ semileptonic decays to nonzero recoil. That would reduce the uncertainty due to the extrapolation to $\omega = 1$; indeed, experimental data need to be taken at $\omega \neq 1$ due to the vanishing phase space at the zero recoil point. At finite momentum transfer, only old quenched lattice results are available [13] which, combined with 2008 BaBar data [14], give $|V_{cb}| = 37.4 \pm 0.5_{\text{exp}} \pm 0.8_{\text{th}}$.

By using zero recoil sum rules, the more recent form factor value obtained is [15, 16]

$$\mathcal{F}(1) = 0.86 \pm 0.02 \quad (6)$$

in good agreement with the lattice value in Eq. (4), but slightly lower in the central value. That implies a relatively higher value of $|V_{cb}|$, that is

$$|V_{cb}| = (41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}) \times 10^{-3} \quad (7)$$

where the HFAG averages have been used. The theoretical error is more than twice the error in the lattice determination (5).

In $B \rightarrow D l \nu$ decay, the form factor has been calculated at all recoils in the unquenched form approximation by the FNAL/MILC collaboration [17], giving the value

$$|V_{cb}| = (38.5 \pm 1.9_{\text{exp+lat}} \pm 0.2_{\text{QED}}) \times 10^{-3} \quad (8)$$

The first error combines statistical and systematic errors from both experiment and theory. The second error reflects the uncertainty in the Coulomb correction. The error could be improved by repeating the analysis with a world average of experimental form factors, and/or by ameliorating the understanding of the experimental systematic error at large ω due to the vanishing phase space. To quantify the improvement due to working at nonzero recoil, $|V_{cb}|$ is also extracted by extrapolating the experimental data to zero recoil and comparing with the theoretical form factor at that point. The result is found consistent with the nonzero recoil determination, within the (expected) larger error [17].

Heavy-quark discretization errors are the largest source of uncertainty on $|V_{cb}|$ determinations by the FNAL/MILC collaboration using both exclusive $B \rightarrow D^* l \nu$ and $B \rightarrow D l \nu$ decays. Work is in progress to reduce them by improving the Fermilab action to third order in HQET [18].

In the alternative lattice approach based on the step scaling method, which avoids the recourse to HQET, the value for the form factor is only available at non-zero recoil in the quenched approximation [19, 20]. By using 2009 data from BaBar Collaboration, for $B \rightarrow D l \nu$ decays, [21], the value $|V_{cb}| = 37.4 \pm 0.5_{\text{exp}} \pm 0.8_{\text{th}}$ is obtained. The errors are statistical, systematic and due to the theoretical uncertainty in the form factor \mathcal{G} , respectively.

On the non-lattice front, the "BPS" limit is the limit where the parameters related to kinetic energy and the chromomagnetic moment are equal in the heavy quark expansion [22]. Using this limit, the Particle Data Group finds the form factor [23]

$$\mathcal{G}(1) = 1.04 \pm 0.02 \quad (9)$$

and the related

$$|V_{cb}| = (40.6 \pm 1.5_{\text{exp}} \pm 0.8_{\text{th}}) \times 10^{-3} \quad (10)$$

In this section, we have always implicitly alluded to B decays, but semileptonic B_s decays can also probe CKM matrix elements. Moreover, semileptonic B_s^0 decays are used as a normalization mode for various searches for new physics at hadron colliders and at Belle-II. On lattice, the valence strange quarks needs less of a chiral extrapolation and is better accessible in numerical simulations with respect to the physical $u(=d)$ -quark. Zero-recoil form factors at $N_f = 2$ have been computed for $B_s \rightarrow D_s l \nu$ decays [24], which is easier involving less form factors than $B_s \rightarrow D_s^* l \nu$ decays.

Inclusive $B \rightarrow X_c l \nu_l$ decays

In inclusive $B \rightarrow X_c l \nu_l$ decays, the final state X_c is an hadronic state originated by the charm quark. There is no dependence on the details of the final state, and quark-hadron duality is generally assumed. Sufficiently inclusive quantities (typically the width and the first few moments of kinematic distributions) can be expressed as a double series in α_s and Λ_{QCD}/m , in the framework of the Heavy Quark Expansion (HQE), schematically indicated as

$$\Gamma(B \rightarrow X_c l \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{qb}|^2 \left[c_3 \langle O_3 \rangle + c_5 \frac{\langle O_5 \rangle}{m_b^2} + c_6 \frac{\langle O_6 \rangle}{m_b^3} + O\left(\frac{\Lambda_{QCD}^4}{m_b^4}, \frac{\Lambda_{QCD}^5}{m_b^3 m_c^2} + \dots\right) \right] \quad (11)$$

Here c_d ($d = 3, 5, 6, \dots$) are short distance coefficients, calculable in perturbation theory as a series in the strong coupling α_s , and O_d denote local operators of (scale) dimension d , whose hadronic expectation values $\langle O_d \rangle$ encode the nonperturbative corrections. The hadronic expectation values of the operators can be parameterized in terms of HQE parameters, whose number grows at increasing powers of Λ_{QCD}/m . These parameters are affected by the particular theoretical framework (scheme) that is used to define the quark masses. Let us observe that the first order in the series corresponds to the parton order, while terms of order Λ_{QCD}/m are absent. At highest orders in Λ_{QCD}/m_b , terms including powers of Λ_{QCD}/m_c have to be considered as well. Indeed, roughly speaking, since $m_c^2 \sim O(m_b \Lambda_{QCD})$ and $\alpha_s(m_c) \sim O(\Lambda_{QCD})$, contributions of order $\Lambda_{QCD}^5/m_b^3 m_c^2$ and $\alpha_s(m_c) \Lambda_{QCD}^4/m_b^3 m_c$ are expected comparable in size to contributions of order Λ_{QCD}^4/m_b^4 .

At order $1/m_b^0$ in the HQE, that is the parton level, the perturbative corrections up to order α_s^2 to the width and to the moments of the lepton energy and hadronic mass distributions are known completely (see Refs. [25, 26, 27, 28, 29] and references therein). The terms of order $\alpha_s^{n+1} \beta_0^n$, where β_0 is the first coefficient of the QCD β function, have also been computed following the BLM procedure [30, 26]. The next order is Λ_{QCD}^2/m_b^2 , and at this order the HQE includes two operators, called the kinetic energy and the chromomagnetic operator. The perturbative corrections to the coefficient of the matrix element of the kinetic operator have been evaluated at order α_s^2 for generic observables, such as partial rates and moments [31, 32]. Corrections at order α_s^2 to the coefficient of the matrix element of the chromomagnetic

TABLE 1. Status of recent inclusive and exclusive $|V_{cb}|$ determinations

Exclusive decays		$ V_{cb} \times 10^3$
$\bar{B} \rightarrow D^* l \bar{\nu}$		
FNAL/MILC (Lattice unquenched) [11]	$39.04 \pm 0.49_{\text{exp}} \pm 0.53_{\text{latt}} \pm 0.19_{\text{QED}}$	
HFAG (Lattice unquenched) [39, 9, 10]	$39.54 \pm 0.50_{\text{exp}} \pm 0.74_{\text{th}}$	
Rome (Lattice quenched $\omega \neq 1$) [13, 14]	$37.4 \pm 0.5_{\text{exp}} \pm 0.8_{\text{th}}$	
HFAG (Sum Rules) [15, 16, 39]	$41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}$	
$\bar{B} \rightarrow D l \bar{\nu}$		
FNAL/MILC (Lattice unquenched $\omega \neq 1$) [17]	$38.5 \pm 1.9_{\text{exp+lat}} \pm 0.2_{\text{QED}}$	
PDG (HQE + BPS) [23, 22]	$40.6 \pm 1.5_{\text{exp}} \pm 0.8_{\text{th}}$	
Rome (Lattice quenched $\omega \neq 1$) [21, 19]	$41.6 \pm 1.8_{\text{stat}} \pm 1.4_{\text{syst}} \pm 0.7_{\text{FF}}$	
Inclusive decays		
kin scheme (HFAG) [39]	42.46 ± 0.88	
kin scheme [40]	42.21 ± 0.78	
Indirect fits		
UTfit [41]	41.7 ± 0.6	
CKMfitter (3σ) [42]	$41.4^{+1.4}_{-1.8}$	

operator have also been completed recently [33, 34]. Let us observe that the latest results in Ref. [34] present slight differences with previous results in Ref. [35].

Neglecting perturbative corrections, i.e. working at tree level, contributions to various observables have been computed at order $1/m_b^3$ [36], $1/m_b^4$ [37] and estimated at order $1/m_b^5$ [35, 38].

A global fit is a simultaneous fit to HQE parameters, quark masses and absolute values of CKM matrix elements obtained by measuring spectra plus all available moments. The HFAG global fit employs as experimental inputs the (truncated) moments of the lepton energy E_l (in the B rest frame) and the m_X^2 spectra in $B \rightarrow X_c l \nu$ [39]. The actual HFAG global fit is performed in the kinetic scheme, includes 6 non-perturbative parameters ($m_{b,c}$, $\mu_{\pi,G}^2$, $\rho_{D,LS}^3$) and the NNLO $O(\alpha_s)$ corrections, yielding

$$|V_{cb}| = (42.46 \pm 0.88) \times 10^{-3} \quad (12)$$

A very recent determination in the kinetic scheme, with a global fit which includes the complete power corrections up to $O(\alpha_s \Lambda_{QCD}^2/m_b^2)$, gives [40]

$$|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3} \quad (13)$$

The two results have practically the same average value, and the uncertainty is about 2% and 1.8%, respectively.

Inclusive and exclusive results have been collected in Table 1. The uncertainty on the inclusive and of the exclusive determinations (from $B \rightarrow D^*$ semileptonic decays) is about 2%, while the uncertainty on the determination from $B \rightarrow D$ semileptonic decays is about 5%. We observe a tension of 2.9σ between the latest FNAL/MILC lattice result [11] and the result from the latest global fit in the inclusive case [40].

It is also possible to determine $|V_{cb}|$ indirectly, using the CKM unitarity relations together with CP violation and flavor data, excluding direct informations on decays. The indirect fit provided by the UTfit collaboration [41] gives

$$|V_{cb}| = (41.7 \pm 0.6) \times 10^{-3} \quad (14)$$

while the CKMfitter collaboration (at 3σ) [42] finds

$$|V_{cb}| = (41.4^{+1.4}_{-1.8}) \times 10^{-3} \quad (15)$$

Indirect fits prefer a value for $|V_{cb}|$ that is closer to the (higher) inclusive determination.

B-Mesons Decays to Excited D-Meson States

The increased interest in semi-leptonic B decays to excited states of the charm meson spectrum derives by the fact that they contribute as a background to the direct decay $B^0 \rightarrow D^* l \nu$ at the B factories, and, as a consequence, as a source of systematic error in the $|V_{cb}|$ measurements.

The spectrum of mesons consisting of a charm and an up or a down anti-quark is poorly known. In the non-relativistic constituent quark model, the open charm system can be classified according to the radial quantum number and to the eigenvalue L of the relative angular momentum between the c-quark and the light degrees of freedom. In the limit where the heavy quark mass is infinity, the spin of the heavy quark is conserved and decouples from the total angular momentum of the light degrees of freedom. The latter, $\vec{j}_l \equiv \vec{L} + \vec{s}_q$, with \vec{s}_q being the spin of the light degrees of freedom, becomes a conserved quantity as well. Of the four states with $L = 1$, $D_1(2420)$ and $D_2^*(2460)$ have relatively narrow widths, about 20-30 MeV, and have been observed and studied by a number of experiments since the nineties (see Ref. [43] and references therein). The other two states, $D_0^*(2400)$, $D_1'(2430)$, are more difficult to detect due to the large width, about 200-400 MeV, and their observation has started more recently [44, 45, 46, 47, 48]. Theoretically, the states with large width correspond to $j_l = 1/2^+$ states, which decay as $D_{0,1}^* \rightarrow D^{(*)} \pi$ through S waves by conservation of parity and angular momentum. Similarly, the states with small width correspond to $j_l = 3/2^+$ states, since $D_2^* \rightarrow D^{(*)} \pi$ and $D_1 \rightarrow D^* \pi$ decay through D waves. To be precise, the $D_1 \rightarrow D^* \pi$ decays may occur a priori through D and S waves, but the latter are disfavored by heavy quark symmetry.

In 2010 BaBar has observed, for the first time, candidates for the radial excitations of the D^0 , D^{*0} and D^{*+} , as well as the $L = 2$ excited states of the D^0 and D^+ [49]. Resonances in the 2.4-2.8 GeV/ c^2 region of hadronic masses have also been identified at LHCb [50].

The not completely clear experimental situation is mirrored by two theoretical puzzles. Most calculations, using sum rules [51, 52], quark models [53, 54, 55, 56], OPE [57, 58] (but not constituent quark models [59]), indicate that the narrow width states dominate over the broad D^{**} states, in contrast to experiments (the “1/2 vs 3/2” puzzle). One possible weakness common to these theoretical approaches is that they are derived in the heavy quark limit and corrections might be large. The other puzzle is that the sum of the measured semi-leptonic exclusive rates having $D^{(*)}$ in the final state is less than the inclusive one (“gap” problem) [48, 60]. Indeed, decays into $D^{(*)}$ make up $\sim 70\%$ of the total inclusive $B \rightarrow X_c l \bar{\nu}$ rate and decays into $D^{(*)} \pi$ make up another $\sim 15\%$, leaving a gap of about 15%. Recently, the full BABAR data set has been used to improve the precision on decays involving $D^{(*)} \pi l \nu$ and to search for decays of the type $D^{(*)} \pi \pi l \nu$. Preliminary results assign about 0.7% to $D^{(*)} \pi \pi l \nu$, reducing the significance of the gap from 7σ to 3σ [61]. Let us also mention that lattice studies are in progress with realistic charm mass, and preliminary results on $\bar{B} \rightarrow D^{**} l \nu$ form factors are available [12, 62, 63].

HEAVY-TO-LIGHT DECAYS

Exclusive decays

The analysis of exclusive charmless semi-leptonic decays, in particular the $\bar{B} \rightarrow \pi l \bar{\nu}_l$ decay, is currently employed to determine the CKM parameter $|V_{ub}|$. The $B \rightarrow \pi l \nu$ decays depend on a single form factor $f_+(q^2)$, in the limit of massless leptons. The first lattice determinations of $f_+(q^2)$ based on unquenched simulations have been obtained by the HPQCD collaboration [64] and the Fermilab/MILC collaboration [65]; they are in substantial agreement. These analyses, at $q^2 > 16 \text{ GeV}^2$, together with latest data on $B \rightarrow \pi l \nu$ decays coming from Belle and BaBar, and 2007 data from CLEO, have been employed in the actual HFAG averages [39]. Also, HFAG has performed a simultaneous fit of the BCL parametrization [66] to lattice results and experimental data, to exploit all the available information in the full q^2 range, which has given the following average value

$$|V_{ub}| = (3.28 \pm 0.29) \times 10^{-3} \quad (16)$$

The Fermilab/MILC collaboration has recently presented an update, based on 12 of the MILC (2+1)-flavor asqtad ensembles, at four different lattice spacings over the range $a \sim 0.045\text{-}0.12 \text{ fm}$, yielding as a preliminary result [67]

$$|V_{ub}| = (3.72 \pm 0.14) \times 10^{-3} \quad (17)$$

where the error reflects both the lattice and experimental uncertainties, which are now on par with each other. Further results on form factors have been presented by the ALPHA [68, 69] ($N_f = 2$) HPQCD [70] ($N_f = 2 + 1$), and the

TABLE 2. Status of recent exclusive $|V_{ub}|$ determinations and indirect fits

Exclusive decays	$ V_{ub} \times 10^3$
$\bar{B} \rightarrow \pi l \bar{\nu}_l$	
HPQCD ($q^2 > 16$) (HFAG) [64, 39]	$3.52 \pm 0.08^{+0.61}_{-0.40}$
Fermilab/MILC ($q^2 > 16$) (HFAG) [65, 39]	$3.36 \pm 0.08^{+0.37}_{-0.31}$
Fermilab/MILC prelim. 2014 [67]	3.72 ± 0.14
lattice, full q^2 range (HFAG) [39]	3.28 ± 0.29
LCSR ($q^2 < 12$) (HFAG) [74, 39]	$3.41 \pm 0.06^{+0.37}_{-0.32}$
LCSR ($q^2 < 16$) (HFAG) [77, 39]	$3.58 \pm 0.06^{+0.59}_{-0.40}$
lattice+ LCSR (Belle) [80]	3.52 ± 0.29
LCSR ($q^2 < 12$) Bayes. an. [79]	$3.32^{+0.26}_{-0.22}$
Indirect fits	
UTfit [41]	3.63 ± 0.12
CKMfitter (at 3σ) [42]	$3.57^{+0.41}_{-0.31}$

RBC/UKQCD [71] ($N_f = 2 + 1$) collaborations. In the quenched approximation, calculations using the $O(\alpha_s)$ improved Wilson fermions and $O(\alpha_s)$ improved currents have been performed on a fine lattice (lattice spacing $a \sim 0.04$ fm) by the QCDSF collaboration [72] and on a coarser one (lattice spacing $a \sim 0.07$ fm) by the APE collaboration [73].

At large recoil, direct LCSR calculations of the semi-leptonic form factors are available, which have benefited by progress in pion distribution amplitudes, next-to-leading and leading higher order twists (see e.g. Refs. [74, 75, 76] and references within). The $|V_{ub}|$ estimate are generally higher than the corresponding lattice ones, but still in agreement, within the relatively larger theoretical errors. The estimated values for $|V_{ub}|$ according to LCSR [77, 74] provided by HFAG have been reported in Table 2. Higher values for $|V_{ub}|$ have been computed in the relativistic quark model [78]. The latest LCSR determination of $|V_{ub}|$ uses a Bayesian uncertainty analysis of the $B \rightarrow \pi$ vector form factor and combined BaBar/Belle data within the framework of LCSR at ($q^2 < 12$), yielding [79]

$$|V_{ub}| = (3.32^{+0.26}_{-0.22}) \times 10^{-3} \quad (18)$$

By using hadronic reconstruction, Belle finds a branching ratio of $\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu) = (1.49 \pm 0.09_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-4}$ [80], which is competitive with the more precise results from untagged measurements. By employing this measured partial branching fraction, and combining LCSR, lattice points and the BCL [66] description of the $f_+(q^2)$ hadronic form factor, Belle extracts the value

$$|V_{ub}| = (3.52 \pm 0.29) \times 10^{-3} \quad (19)$$

This value is also reported in Table 2, where it is also compared with indirect fits, that is with

$$|V_{ub}| = (3.63 \pm 0.12) \times 10^{-3} \quad (20)$$

given by UTfit Collab, [41] and with

$$|V_{ub}| = (3.57^{+0.41}_{-0.31}) \times 10^{-3} \quad (21)$$

given (at 3σ) by CKMfitter [42].

Recently, significantly improved branching ratios of other heavy-to-light semi-leptonic decays have been reported, that reflect on increased precision for $|V_{ub}|$ values inferred by these decays. The $B^+ \rightarrow \omega l^+ \nu$ branching fraction has been measured by the Babar collaboration with semileptonically tagged B mesons [81]. The value of $|V_{ub}|$ has been extracted from $B^+ \rightarrow \omega l^+ \nu$ [81], yielding, with the LCSR form factor determination [82]

$$|V_{ub}| = (3.41 \pm 0.31) \times 10^{-3} \quad (22)$$

and, with the ISGW2 quark model[83]

$$|V_{ub}| = (3.43 \pm 0.31) \times 10^{-3} \quad (23)$$

TABLE 3. Status of recent inclusive $|V_{ub}|$ determinations

Inclusive decays ($V_{ub} \times 10^3$)				
	BNLP [103, 104, 105]	GGOU [106]	ADFR [107, 108, 109]	DGE [110]
BaBar [102]	$4.28 \pm 0.24^{+0.18}_{-0.20}$	$4.35 \pm 0.24^{+0.09}_{-0.10}$	$4.29 \pm 0.24^{+0.18}_{-0.19}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$
Belle [101]	$4.47 \pm 0.27^{+0.19}_{-0.21}$	$4.54 \pm 0.27^{+0.10}_{-0.11}$	$4.48 \pm 0.30^{+0.19}_{-0.19}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$
HFAG [39]	$4.40 \pm 0.15^{+0.19}_{-0.21}$	$4.39 \pm 0.15^{+0.12}_{-0.20}$	$4.03 \pm 0.13^{+0.18}_{-0.12}$	$4.45 \pm 0.15^{+0.15}_{-0.16}$

A major problem is that the quoted uncertainty does not include any uncertainty from theory, since uncertainty estimates of the form-factor integrals are not available.

The Babar collaboration has also investigated the $B \rightarrow \rho l \nu$ channel [84]. By comparing the measured distribution in q^2 , with an upper limit at $q^2 = 16$ GeV, for $B \rightarrow \rho l \nu$ decays, they obtain [84], (with LCSR predictions for the form factors [82])

$$|V_{ub}| = (2.75 \pm 0.24) \times 10^{-3} \quad (24)$$

and with the ISGW2 quark model[83].

$$|V_{ub}| = (2.83 \pm 0.24) \times 10^{-3} \quad (25)$$

More recent results on both $B \rightarrow \omega l \nu$ decays and $B \rightarrow \rho l \nu$ decays have been presented by a Belle tagged analysis [80]. In the same analysis [80], an evidence of a broad resonance around 1.3 GeV dominated by the $B^+ \rightarrow f_2 l \nu$ decay has also been reported for the first time.

The branching fractions for $B \rightarrow \eta^{(\prime)} l \nu$ decays have been measured by the BaBar collaboration [85]. The value of the ratio

$$\frac{\mathcal{B}(B^+ \rightarrow \eta' l^+ \nu_l)}{\mathcal{B}(B^+ \rightarrow \eta l^+ \nu_l)} = 0.67 \pm 0.24_{\text{stat}} \pm 0.11_{\text{syst}} \quad (26)$$

seems to allow an important gluonic singlet contribution to the η' form factor [85, 86]. In future prospects, other channels that can be valuable to extract $|V_{ub}|$ are $B_s \rightarrow K^{(*)} \bar{l} \nu$ decays [87, 88, 89]. Let us also mention the baryonic semileptonic $\Lambda_b^0 \rightarrow p l^- \bar{\nu}$ decays, which depends on $|V_{ub}|$ as well [90, 91, 92].

Inclusive $B \rightarrow X_u l \nu_l$ decays

The extraction of $|V_{ub}|$ from inclusive decays requires to address theoretical issues absent in the inclusive $|V_{cb}|$ determination. OPE techniques are not applicable in the so-called endpoint or singularity or threshold phase space region, corresponding to the kinematic region near the limits of both the lepton energy E_l and q^2 phase space, where the rate is dominated by the production of low mass final hadronic states. This region is plagued by the presence of large double (Sudakov-like) perturbative logarithms at all orders in the strong coupling. Corrections can be large and need to be resummed at all orders². The kinematics cuts due to the large $B \rightarrow X_c l \nu$ background enhance the weight of the threshold region with respect to the case of $b \rightarrow c$ semi-leptonic decays; moreover, in the latter, corrections are not expected as singular as in the $b \rightarrow u$ case, being cutoff by the charm mass.

On the experimental side, efforts have been made to control the background and access to a large part of the phase space, so as to reduce, on the whole, the weight of the endpoint region. Latest results by Belle [101] and BaBar [102] use their complete data sample, 657×10^6 $B\bar{B}$ pairs for Belle and 467×10^6 $B\bar{B}$ pairs for BaBar. Although the two analyses differ in the treatment of the background, both collaborations claim to access $\sim 90\%$ of the phase space.

On the theoretical side, several schemes are available. All of them are tailored to analyze data in the threshold region, but differ significantly in their treatment of perturbative corrections and the parametrization of non-perturbative effects.

The analyses from BaBar [102] and Belle [101] collaborations, as well as the HFAG averages [39], rely on at least four theoretical different QCD calculations of the inclusive partial decay rate: BNLP by Bosch, Lange, Neubert, and Paz [103, 104, 105]; GGOU by Gambino, Giordano, Ossola and Uraltsev [106]; ADFR by Aglietti, Di Lodovico, Ferrara, and Ricciardi [107, 108, 109]; DGE, the dressed gluon exponentiation, by Andersen and Gardi [110]. They can

² See e.g. Refs. [93, 94, 95, 96, 97, 98, 99, 100] and references therein.

be roughly divided into approaches based on the estimation of the shape function (BLNP, GGOU) and on resummed perturbative QCD (ADFR, DGE). Although conceptually quite different, all the above approaches generally lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. The HFAG estimates [39], together with the latest estimates by BaBar [102, 111] and Belle [101], are reported in Table 3. The BaBar and Belle estimates in Table 3 refers to the value extracted by the most inclusive measurement, namely the one based on the two-dimensional fit of the $M_X - q^2$ distribution with no phase space restrictions, except for $p_l^* > 1.0$ GeV. This selection allow to access approximately 90% of the total phase space [111]. The BaBar collaboration also reports measurements of $|V_{ub}|$ in other regions of the phase space [102], but the values reported in Table 3 are the most precise. The arithmetic average of the results obtained from these four different QCD predictions of the partial rate gives [102]

$$|V_{ub}| = 4.33 \pm 0.24_{\text{exp}} \pm 0.15_{\text{th}} \quad (27)$$

By comparing this result (or results in Table 3) with results in Table 2, we observe a tension between exclusive and inclusive determinations, of the order of 3σ . At variance with the $|V_{cb}|$ case, the results of the global fit prefer a value for $|V_{ub}|$ that is closer to the (lower) exclusive determination. A lot of theoretical effort has been devoted to clarify the present tension by inclusion of NP effects. A recent claim excludes the possibility of a NP explanation of the difference between the inclusive and exclusive determinations of $|V_{ub}|$ [112].

τ LEPTONS IN THE FINAL STATE

Semileptonic decays

The $B \rightarrow D^{(*)} \tau \nu_\tau$ decays are more difficult to measure, since decays into the heaviest τ lepton are suppressed and there are multiple neutrinos in the final state, following the τ decay. Multiple neutrinos stand in the way of the reconstruction of the invariant mass of B meson, and additional constraints related to the B production are required. At the B factories, a major constraint exploited is the fact that B mesons are produced from the process $e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$.

The BaBar Collaboration has measured the $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ branching fractions normalized to the corresponding $\bar{B} \rightarrow D^{(*)} l^- \bar{\nu}_l$ modes (with $l = e, \mu$) by using the full BaBar data sample, and found [113, 114]

$$\begin{aligned} \mathcal{R}_{\tau/l}^* &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^* \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^* l^- \bar{\nu}_l)} = 0.332 \pm 0.024 \pm 0.018 \\ \mathcal{R}_{\tau/l} &\equiv \frac{\mathcal{B}(\bar{B} \rightarrow D \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D l^- \bar{\nu}_l)} = 0.440 \pm 0.058 \pm 0.042 \end{aligned} \quad (28)$$

where the first uncertainty is statistical and the second is systematic. The results exceed the SM predictions $\mathcal{R}_{\tau/l}^*(SM) = 0.252 \pm 0.003$ and $\mathcal{R}_{\tau/l}(SM) = 0.297 \pm 0.017$ by 2.7σ and 2.0σ , respectively. The combined significance of this disagreement is 3.4σ [113, 114]. In the case of $\mathcal{R}_{\tau/l}$, the SM prediction has been revisited with different approaches: a combined phenomenological and lattice analysis [115] yields $\mathcal{R}_{\tau/l}(SM) = 0.31 \pm 0.02$, and a similar result, $\mathcal{R}_{\tau/l}(SM) = 0.316 \pm 0.012 \pm 0.007$, where the errors are statistical and total systematic, respectively, is found in a (2+1)-flavor lattice QCD calculation [116]. Both SM analysis reduce the significance of the discrepancy for $\mathcal{R}_{\tau/l}$ below 2σ .

The BaBar results (28) are in agreement (with smaller uncertainties) with measurements by Belle using the $\Upsilon(4S)$ data set that corresponds to an integrated luminosity of 605 fb^{-1} and contains $657 \times 10^6 B\bar{B}$ events [117]. The branching ratio measured values by the two beauty factories have consistently exceeded the SM expectations since 2007, but now the increased precision starts to be enough to constrain NP. The latest data from BaBar are not compatible with a charged Higgs boson in the type II two-Higgs-doublet model and with large portions of the more general type III two-Higgs-doublet model [114]. The alleged breaking of lepton-flavour universality suggested by data is quite large, of the order of 30%, and several theoretical models have been tested against the experimental results: minimal flavor violating models, right-right vector and right-left scalar quark currents, leptoquarks, quark and lepton compositeness models have been investigated [118, 119], modified couplings [120, 121], additional tensor operators [122], charged scalar contributions [123], effective Lagrangians [124, 121], new sources of CP violation [125], and so on. The A2HDM does not seem able to accomodate present data on $\mathcal{R}_{\tau/l}$ [126].

There is room for improvement in current statistic limits for measurements of $\mathcal{B}_{\tau/l}$. It would be interesting to ascertain if the results of the Belle analysis will shift towards the SM predictions by using the full $Y(4S)$ data sample containing 772×10^6 $B\bar{B}$ pairs and the improved hadronic tagging, as happened in the case of purely leptonic decays $B^- \rightarrow \tau^- \bar{\nu}_\tau$ [127].

At Belle II, with more data, there will be a better understanding of backgrounds tails under the signal. At 5 ab^{-1} the expected uncertainty is of 3% for $\mathcal{B}_{\tau/l}^*$ and 5% for $\mathcal{B}_{\tau/l}$. Data from Belle II may in principle be used for the inclusive $B \rightarrow X_c \tau \nu$ decays, where recent predictions for the dilepton invariant mass and the τ energy distributions have been performed [128].

Leptonic decays

In the SM, the purely leptonic decay $B \rightarrow l \nu_l$ has the branching ratio

$$\mathcal{B}(B \rightarrow l \nu_l) = \frac{G_F^2 m_B m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \quad (29)$$

The only charged current B meson decay that has been observed so far is the $B \rightarrow \tau \nu_\tau$ decay, which was observed for the first time by Belle in 2006 [129]. Its measurement provides a direct experimental determination of the product $f_B |V_{ub}|$. The decay constant f_B parameterizes the matrix elements of the axial vector current

$$\langle 0 | \bar{b} \gamma^\mu \gamma_5 q | B_q(p) \rangle = p_B^\mu f_B \quad (30)$$

For heavy-light mesons, it is sometimes convenient to define and study the quantity

$$\Phi_B \equiv f_B \sqrt{m_B} \quad (31)$$

which approaches a constant (up to logarithmic corrections) in the $m_B \rightarrow \infty$ limit. The branching fractions for the $B \rightarrow \tau \nu_\tau$ decays have been measured by the Belle and BaBar collaborations, with uncertainties dominated by statistical errors, and individual significance below 5σ . When combined, they cross the threshold needed to establish discovery in this mode. Until recently, all the measurements were in agreement within the errors; the HFAG average yields [39]

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (1.67 \pm 0.30) \times 10^{-4} \quad (32)$$

which is nearly 3σ higher than the SM estimate based on a global fit. However, the latest Belle measurement [127] obtains a result which is more than two σ below the previous averages

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (0.72_{-0.25}^{+0.27} \pm 0.11) \times 10^{-4} \quad (33)$$

where the first errors are statistical and the second one systematical. This is the single-most precise determination of the $B \rightarrow \tau \nu_\tau$ branching fraction, obtained using the hadronic tagging method with the full dataset. By using this Belle value together with the previous Belle measurements based on a semi-leptonic B tagging method and taking into account all the correlated systematic errors, the Belle branching fraction becomes [127]

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (0.96 \pm 0.26) \times 10^{-4} \quad (34)$$

In contrast with previous experimental analyses, the new Belle data seem to indicate agreement with the SM results.

Combining the value (34) with the mean B^+ -meson lifetime $\tau_B = 1.641 \pm 0.008$ [23] and their average for the B meson decay constant, $f_B = 190.5 \pm 4.2 \text{ MeV}$ ($N_f = 2 + 1$), the FLAG (Flavor Lattice Averaging Group) collaboration obtains [130]

$$|V_{ub}| = (3.87 \pm 0.52 \pm 0.09) \times 10^{-3} \quad (35)$$

where the first error comes from experiment and the second comes from the uncertainty in f_B . The FLAG collaboration also presents an average of Belle and BaBar results, neglecting the correlation between systematic errors in the measurements obtained using the hadronic and semileptonic tagging. They obtain [130]

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau) = (1.12 \pm 0.28) \times 10^{-4} \quad (36)$$

where a rescaling factor $\sqrt{\chi^2/\text{d.o.f.}} \sim 1.3$ has been applied to take into account the fact that the Belle hadronic tagging measurements differ significantly from the others. By using this value for the branching fraction, and combining with their lattice-QCD average for f_B , the FLAG collaboration obtains, in the $N_f = 2 + 1$ case,

$$|V_{ub}| = (4.18 \pm 0.52 \pm 0.09) \times 10^{-3} \quad (37)$$

and

$$|V_{ub}| = (4.28 \pm 0.53 \pm 0.09) \times 10^{-3} \quad (38)$$

in the $N_f = 2 + 1 + 1$ case. The average values seem to point towards the semileptonic inclusive $|V_{ub}|$ determinations, as can be seen by comparison with the values in Table 3. The accuracy is not yet enough to make the leptonic channel competitive for $|V_{ub}|$ extraction. Finally, let us just mention that search of possible lepton flavour violations can also be made independently of $|V_{ub}|$ by building ratios of branching fractions, such as $R' = \tau_{B^0}/\tau_{B^+} \mathcal{B}(B^+ \rightarrow \tau^+ \nu_\tau)/\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu_l)$.

CONCLUSIONS

We are experiencing a period of impressive experimental progress. Just to mention a few recent developments: BaBar and Belle have pushed experimental cuts on inclusive semileptonic $B \rightarrow X_u l \bar{\nu}$ decays so far to cover about 90% of the available phase space, preliminary findings by BaBar seem on their way to solve the long standing gap puzzle for $B \rightarrow D^{(*)} l \bar{\nu}$ decays, higher and higher precision is being achieved in measurements of exclusive $B \rightarrow \rho/\omega l \bar{\nu}$ decays as well as of semileptonic and leptonic decays with a final τ . More interesting results are expected, at present from further analyses of data provided by the beauty factories and from LHCb, and in the (approaching) future from Belle II. SuperKEKB construction is on schedule and will start commissioning at the beginning of 2015. Physics run is anticipated to start in 2017.

Progress have also been registered on the theoretical side, and the situation is rich in perspective. The perturbative calculations, in general, have reached a phase of maturity, and the larger theoretical errors are due to non-perturbative approaches. Errors have been recently lowered in both lattice and LCSR frameworks; new global fits for inclusive processes also sport further reduced theoretical uncertainties. New physics is always more constrained. Still awaiting firmly established solutions are a few dissonances within the SM, such as the so-called “1/2 vs 3/2” and “gap” puzzles, the possibility of flavour violation observed in decays into tauons, and the tension between the inclusive and exclusive determination of $|V_{cb}|$ and $|V_{ub}|$. The present uncertainty on both the inclusive and the exclusive determinations (from $B \rightarrow D^*$ semileptonic decays) of $|V_{cb}|$ is about 2%, while the uncertainty on the determination from $B \rightarrow D$ semileptonic decays is about 5%. The parameter $|V_{ub}|$ is the less precisely known among the modules of the CKM matrix elements. The error on the inclusive determinations, around 4%, is about one half than the one on the exclusive determinations, which ranges around 8-9%.

Belle II is expected, within the next decade, to roughly halve experimental errors on both inclusive and exclusive $|V_{ub}|$ determinations. Most promising are exclusive analysis of with hadronic tags. In the long run, at about 50 ab^{-1} , the experimental error on the exclusive determinations is expected to become of order 1 – 2%, and smaller than the error on the inclusive determinations.

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